

嶺東技術學院

九十一學年度碩士班入學招生考試試題

統計學

一、 Let $f(x, y)$ be the joint probability density function of two discrete random variables X and Y . The $f(x, y)$ is given by the following table

$f(x, y)$	x		
	0	1	2
0	0.10	0.20	0.10
y			
1	0.15	0.25	0.20

1. Find the variance of X and Y , respectively. (4 分)
2. Find the covariance of X and Y . (3 分)
3. Find the variance of $3X+2Y$. (4 分)
4. Find the conditional probability distribution of Y , given $X=x$. (5 分)
5. Find the conditional expectation of Y , given $X=1$. (To find $E(Y|X=1)$) (3 分)

二、 A survey was conducted in two cities to determine voters support for three parties in an upcoming election 500 qualified voters were randomly selected from each city and the following data were recorded.

Voter support	City I	City II
Party A	204	225
Party B	211	198
Party C	85	77

1. Set up the hypotheses to test whether the 3 parties get the same support between the two cities. (5 分)
2. Conduct an appropriate test at 0.1 significance level. What is your conclusion? (10 分)

三、A random sample X_1, X_2, \dots, X_n of size n is taken from a Bernoulli distribution with an unknown parameter P , $0 < P < 1$.

1. Show that the maximum likelihood estimator for P is $\hat{P} = \frac{\sum_{i=1}^n X_i}{n}$. (10 分)

2. Find the expectation and variance of $\hat{P} = \frac{\sum_{i=1}^n X_i}{n}$, respectively. (6 分)

四、想了解過去三年國內基金及股市資料之報酬率，於是由國內基金及國內股市之過去三年資料中分別抽取 $n_1=11$ 個國內基金及 $n_2=20$ 個國內股市資料，經計算其平均報酬率分別為 $\bar{x}_1=14.8\%$, $\bar{x}_2=9.5\%$ ，且其標準差分別為 $s_1=4.2\%$ 及 $s_2=3.6\%$ 。假設國內基金及國內股市之報酬率皆為近似常態分配，試問

1. 在顯著水準 $\alpha=0.05$ 下，檢定國內基金及國內股市之報酬率風險(標準差)是否相等? (5 分)
2. 在顯著水準 $\alpha=0.05$ 下，檢定國內基金之報酬率是否大於國內股市之報酬率? (5 分)
3. 國內基金及國內股市平均報酬率差 $(\mu_1 - \mu_2)$ 之 95% 的信賴區間(confidence interval)為何? (5 分)

五、1. 請敘述中央極限定理(Central Limit Theorem)? (10 分)

2. 假設嶺東技術學院二技之同學平均每星期上圖書館之時間為 12 小時，且標準差為 4 小時，請問假設隨機抽出 100 名二技學生，該批學生平均每星期上圖書館時數介於 11 至 13 小時之機率為何? (5 分)

六、設有九個地區之放射線的暴露指標及癌症死亡率，其資料之記錄如下：

地區:	1	2	3	4	5	6	7	8	9
暴露指標 X:	8.34	6.41	3.41	3.83	2.57	11.64	1.26	2.49	1.92
死亡率 Y:	210.3	177.9	129.9	162.3	130.1	207.5	113.5	147.1	137.5

(每十萬人每年死亡人數) 已知:

$$\sum X = 41.56, \sum Y = 1416.1, \sum X^2 = 289.422, \sum Y^2 = 232498.97,$$

$$\sum (X - \bar{X})^2 = 97.5074, \sum (X - \bar{X})(Y - \bar{Y}) = 900.13, \sum (Y - \bar{Y})^2 = 9683.50$$

1. 試求以最小平方法所建立之簡單直線迴歸方程式。(4 分)
2. 試計算 X 與 Y 之間的樣本相關係數。(3 分)
3. 若 $X=5.2$ 時，則 Y 之預測值為多少? (3 分)
4. 請檢定母體相關係數 ρ 是否為 0，在 $\alpha=0.05$ 之下。(5 分)
5. 請給與簡單迴歸模式分析之假設。(5 分)

註: 查表之機率為右尾之表示法

1. 標準常態分配:

$$Z_{0.05}=1.645, Z_{0.025}=1.96, Z_{0.01}=2.33, Z_{0.1}=1.282,$$
$$P\{Z>2.12\}=0.0170, P\{Z<2.5\}=0.9938, P\{Z<3\}=0.9987$$
$$P\{-1<Z<1\}=0.6826, P\{-2<Z<2\}=0.9544, P\{-3<Z<3\}=0.9973$$

2. 卡方分配:

$$\chi_{0.05}^2(4)=9.488, \chi_{0.025}^2(4)=11.14, \chi_{0.05}^2(5)=11.07, \chi_{0.025}^2(5)=12.83$$
$$\chi_{0.05}^2(1)=3.8415, \chi_{0.025}^2(1)=5.024, \chi_{0.05}^2(2)=5.991, \chi_{0.025}^2(2)=7.378$$
$$\chi_{0.05}^2(3)=7.8147, \chi_{0.025}^2(3)=9.3484, \chi_{0.1}^2(2)=4.6052,$$
$$\chi_{0.1}^2(3)=6.2514, \chi_{0.1}^2(4)=7.7794$$

3. Student(或 t)分配:

$$t_{0.05}(6)=1.943, t_{0.05}(7)=1.895, t_{0.05}(8)=1.860, t_{0.05}(9)=1.833$$
$$t_{0.025}(6)=2.447, t_{0.025}(7)=2.365, t_{0.025}(8)=2.306, t_{0.025}(9)=2.262$$
$$t_{0.05}(26)=1.7056, t_{0.05}(27)=1.7033, t_{0.05}(28)=1.7011, t_{0.05}(29)=1.6911$$
$$t_{0.025}(26)=2.0555, t_{0.025}(27)=2.0518, t_{0.025}(28)=2.0484, t_{0.025}(29)=2.0452$$

4. F 分配:

$$F_{0.05}(3,5)=5.4099, F_{0.05}(2,6)=5.1432, F_{0.05}(3,6)=4.7571, F_{0.05}(2,7)=4.7374$$
$$F_{0.025}(3,5)=7.7636, F_{0.025}(2,6)=7.2599, F_{0.025}(3,6)=6.5988, F_{0.025}(2,7)=6.5415$$
$$F_{0.05}(1,18)=4.4139, F_{0.05}(2,18)=3.5546, F_{0.05}(10,19)=2.3779, F_{0.05}(19,10)=2.786$$
$$F_{0.025}(1,18)=5.9781, F_{0.025}(2,18)=4.5597, F_{0.025}(10,19)=2.8172,$$
$$F_{0.025}(19,10)=3.436$$

5. 二項分配: 令 X 服從二項分配 B(n; P)

(1) $P\{X \leq 2\}=0.0021, P\{X \leq 3\}=0.0106, P\{X \leq 4\}=0.0384$, 當 $n=16$ 及 $P=0.5$

(2) $P\{X \leq 2\}=0.0012, P\{X \leq 3\}=0.0064, P\{X \leq 4\}=0.0245$, 當 $n=17$ 及 $P=0.5$

(3) $P\{X \leq 2\}=0.0007, P\{X \leq 3\}=0.0038, P\{X \leq 4\}=0.0154$, 當 $n=18$ 及 $P=0.5$

(試題結束)